INTERACTION OF A DISPERSED LIQUID FLOW WITH A HEATED SURFACE

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A study is made of certain features of the interaction of a flow of dispersed liquid with a heated surface in the dry regime and under conditions attending a transition to the wet regime.

The interaction of a dispersed liquid with a heated surface plays an important role in many areas of technology, particularly rocketry, cryogenics, heat treatment of materials, nuclear reactor engineering, etc.

It has been established experimentally [1, 2] that this process may take place in different regimes: in one regime, there is direct contact between the liquid droplet and heated surface; in another, the interaction occurs through a vapor interlayer. The regime in which the interaction takes place determines such important characteristics of the interaction process as the rate of heat exchange between the heated surface and liquid.

This article investigates certain features of the interaction of a dispersed liquid flow with a heated surface in the dry regime (no direct contact between droplet and surface) and under conditions whereby there is a transition to the wet regime. The study was based on a model of a dual-velocity continuum [3]. The following basic parameters were used to describe the state of the system: n, density of droplets; m, mass of droplet (or r, radius of droplet); v_k, velocity of droplet; ρ, density of gas; v, velocity of gas; T, temperature of gas. In the case where these parameters are functions of time t and a single spatial coordinate x, they satisfy the system of differential equations

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv_k)=0,$$

$$\frac{\partial}{\partial t}(nm)+\frac{\partial}{\partial x}(nmv_k)=-nJ,$$

$$\frac{\partial}{\partial t}(nmv_k)+\frac{\partial}{\partial x}(nmv_k^2)=-n(Jv_k+f),$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v)=nJ,$$

$$\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho v^2+p)=n(Jv_k+f),$$

$$\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2}+e\right)+\frac{\partial}{\partial x}\left[v\left(\frac{\rho v^2}{2}+e\right)+pv+q\right]=nJ(-\varkappa+c_pT_s), \tag{1}$$

$x \in [0, L]$, while $x = L$ corresponds to the surface of a wall.

The following relations were used for the gas pressure p, thermal energy density e, heat flux q, rate of droplet evaporation J, and force of interaction between the droplets and gas f

$$p=\rho RT,\quad e=\rho c_V T,\quad q=-\lambda\frac{\partial T}{\partial x},\quad J=4\pi r(\lambda/\varkappa)(T-T_s),$$
$$f=6\pi\eta r(v_k-v). \tag{2}$$

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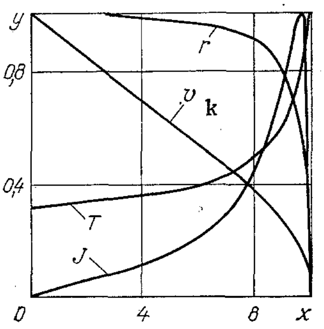

Fig. 1

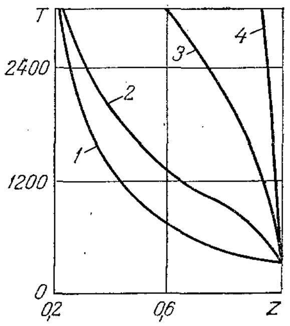

Fig. 2

Fig. 1. Distribution of parameters along x coordinate. Substance Freon-113; $p_0 = 10^5$ Pa; $r_0 = 2\cdot 10^{-5}$ m; $v_{k0} = 10$ m/sec; $h_0 = 4\cdot 10^{-11}$ N; L = 0.1 m; $T_L = 1050$°K; x is the distance, 10^{-2} m; y represents values of the parameters in relative units (y = 1 corresponds to the maximum value of the parameter).

Fig. 2. Dependence of temperature of transition from dry to wet regime on z and h_0. Freon-113; $p_0 = 10^5$ Pa; 1) $h_0 \leqslant 10^{-12}$ N; 2) $4\cdot 10^{-11}$; 3) $4\cdot 10^{-10}$; 4) $4\cdot 10^{-9}$; T_L, °K.

System of equations (1) may be partially integrated for steady-state conditions. We obtain as a result the system of ordinary differential equations

$$\frac{dm}{dx} = -J/v_k, \quad \frac{dv_k}{dx} = -f/mv_k, \quad \frac{dT}{dx} = -q/\lambda \tag{3}$$

and the auxiliary relations

$$n = n_0 v_{k0}/v_k; \tag{4}$$

$$v = \sqrt{b^2 - RT} - b, \quad b = (p_L/nmv_k - v_k)/2; \tag{5}$$

$$p_L = p_0 + n_0 m_0 v_{k0}(v_0 + v_{k0}); \tag{6}$$

$$v_0 = -n_0 m_0 v_{k0} RT_0/p_0; \tag{7}$$

$$\rho = nmv_k/|v|; \tag{8}$$

$$q = q_0 + n_0 v_{k0}[m(\varkappa + c_p(T - T_s) + v^2/2) - m_0(\varkappa + c_p(T_0 - T_s) + v_0^2/2)], \tag{9}$$

the subscripts 0 and L denote values of the parameters at x = 0 and x = L, respectively.

For the pressure p, we obtain from Eq. (1) the following relation

$$\frac{p_L - p}{p} = \beta\left(\beta + \frac{v_K}{v_*}\right), \tag{10}$$

where $\beta = -v/v_*$; $v_* = \sqrt{RT}$ is a value of the order of the speed of sound. Usually, $|v| \ll v_*$ and $|v_k| \ll v_*$. Then it follows from Eq. (10) that $p \approx$ const on the segment [0, L]. This conclusion is confirmed by the numerical results.

Analysis of the system (3)-(9) shows (Appendix A) that, depending on the values of the initial parameters of the system, two qualitatively different dry subregimes of interaction of a dispersed liquid flow with a heated surface are possible. Thus, at

$$T_L \leqslant T_*, \tag{11}$$

where

$$T_* = 3\eta\varkappa/2\lambda, \tag{12}$$

and T_L being the temperature of the wall surface, the droplets are stopped as a result of drag and are not able to completely evaporate ($m > 0$ at $v_k = 0$). If the following conditions are satisfied

$$\frac{p_0}{RT_0}\left(\frac{T_0-T_s}{T_*}-1\right) > n_0 m_0 \text{ and } \frac{T_0-T_s}{T_*} > 1, \tag{13}$$

then the droplets will evaporate completely during their movement (the mass of the droplets m vanishes simultaneously with their velocity v_k).

We numerically integrated the system of equations (3) for different values of the initial parameters of the system. The calculations were performed for Freon-113. The thermophysical properties were taken from [4]. Figure 1 shows a typical distribution of the parameters along the x coordinate.

The results of the calculations were used to determine the dependence of the wall surface temperature T_L characterizing the dry-to-wet-regime transition in the dispersed-flow interaction on other parameters of the system, specifically: initial drop radius r_0, initial drop velocity v_{k0}, initial density of drops n_0, and initial distance to the wall surface L. It turns out that the transition temperature is unambiguously defined by two quantities:

$$z = 6\pi\eta\,\frac{r_0 L}{m_0 v_{k0}} \tag{14}$$

and

$$h_0 = n_0 m_0 r_0^2 v_{k0}^2\,. \tag{15}$$

The function $T_L(z, h_0)$, corresponding to the boundary of the dry regime, is shown in Fig. 2. Curve 1, corresponding to the case $h_0 \to 0$, was obtained analytically (Appendix B) and has the equation

$$T_L = T_s + 3T_*(1/z - 1). \tag{16}$$

This curve nearly coincides with the curves obtained from numerical calculations for $h_0 \leqslant 10^{-12}$ N.

APPENDIX A. CONDITIONS OF REALIZATION OF DIFFERENT DRY SUBREGIMES

We will prove that it follows from the chosen model of interaction of a dispersed liquid flow with a heated surface that the dry regime can be further differentiated into two qualitatively different subregimes.

It follows from (2) and (3) that

$$\frac{dv_k}{dm} = \frac{3\eta\varkappa}{2\lambda}\cdot\frac{v_k - v}{m(T-T_s)}\,.$$

Using Eq. (8) for v and introducing the notation

$$T_* = \frac{3\eta\varkappa}{2\lambda}\,, \tag{A1}$$

we obtain

$$\frac{dv_k}{dm} = \frac{T_*}{T-T_s}\left(\frac{v_k}{m} + \frac{n_0 v_{k0}}{\rho}\right). \tag{A2}$$

Let us convert to polar coordinates in the plane (m, v_k):

$$m = \delta\cos\varphi, \quad v_k = \delta\sin\varphi. \tag{A3}$$

We obtain from (A2) and (A3)

$$\frac{d\delta}{d\varphi} = \delta\,\frac{\mathrm{tg}^2\varphi + \xi\,\mathrm{tg}\,\varphi + a}{(1-a)\,\mathrm{tg}\,\varphi + \xi} \equiv \delta\psi(\varphi), \tag{A4}$$

where $a = (T - T_s)/T_*$; $\xi = (n_o v_{ko})/\rho$.

We will prove that if

$$T_L \leqslant T_*, \tag{A5}$$

then

$$\lim_{x \to x_v} \delta(\varphi(x)) = \delta_v > 0, \tag{A6}$$

where x_v is the point at which v_k vanishes.

Given condition (A5), $a \leqslant T_L/T_* \leqslant 1$. Then it follows from (A4) that $d\delta/d\varphi > 0$. By virtue of (2)-(9)

$$\frac{d\delta}{dx} < 0 \tag{A7}$$

and, thus, $d\varphi/dx < 0$, i.e.,

$$\varphi(x) < \varphi_0 \quad \text{for} \quad x > 0. \tag{A8}$$

Integration of (A4) gives us

$$\delta = \delta_0 \exp\left[-\int_{\varphi}^{\varphi_0} \psi(\varphi)\,d\varphi\right]. \tag{A9}$$

Obviously, there exists a constant M such that

$$0 \leqslant \psi(\varphi) \leqslant M, \quad \varphi \in [0,\ \varphi_0]. \tag{A10}$$

Then it follows from (A8)-(A10) that

$$\delta \geqslant \delta_0 \exp(-M\varphi_0) > 0.$$

Equation (A6) is proven. We thus have proven that if $T_L \leqslant T_*$, then the mass of the droplet $m > 0$ for the entire time of its movement, including the moment it is stopped (when the velocity of the droplet $v_k \to 0$ and $\varphi \to 0$). The value of T_* is determined solely by the thermophysical properties of the liquid and its vapor [Eq. (A1)].

Let us now prove that if

$$\frac{p_0}{RT_0}\left(\frac{T_0 - T_s}{T_*} - 1\right) > n_0 m_0 \ \text{and}\ \frac{T_0 - T_s}{T_*} > 1, \tag{A11}$$

then the mass of the droplet m vanishes simultaneously with its velocity v_k.

Conditions (A11) are equivalent to

$$\mathrm{tg}\,\varphi_0 > b_0 \ \text{and}\ a_0 > 1, \tag{A12}$$

where $b(x) = \xi(x)/(a(x) - 1)$. By virtue of (A4) and (A7)

$$\frac{d\varphi}{dx} = \frac{d\varphi}{d\delta}\,\frac{d\delta}{dx} = \frac{1-a}{\delta(\mathrm{tg}^2\varphi + \xi\,\mathrm{tg}\,\varphi + a)} \cdot (\mathrm{tg}\,\varphi - b)\,\frac{d\delta}{dx}$$

or

$$\frac{d\varphi}{dx} = F(x)\,\Phi(x), \tag{A13}$$

where

$$\Phi(x) = \mathrm{tg}\,\varphi - b, \tag{A14}$$

and $F(x) > 0$.

In accordance with condition (A12), $\Phi(0) > 0$, so that

$$\left.\frac{d\varphi}{dx}\right|_{x=0} > 0. \tag{A15}$$

It follows from (3) and (10) that $db/dx < 0$. Then $b \leqslant b_0$ and

$$\Phi(x) \geqslant \operatorname{tg}\varphi - b_0. \tag{A16}$$

We will prove that

$$\varphi(x) > \varphi_0, \quad x > 0. \tag{A17}$$

Let us assume that there exists a value $x_1 > 0$ such that

$$\varphi(x_1) \leqslant \varphi_0. \tag{A18}$$

Then, by virtue of (A15) and the continuity of $\varphi(x)$, we find a point x_2 such that $\varphi(x_2) = \varphi_0$ and $\varphi(x) > \varphi_0$ for all $x \in (0, x_2)$. Then, from (A13), (A15), and (A16), $d\varphi/dx > 0$, $x \in (0, x_2)$.

On the other hand, according to Lagrange's theorem, there is the point

$$x_3 \in (0,\ x_2), \text{ such that } \left.\frac{d\varphi}{dx}\right|_{x=x_3} = 0.$$

Thus, assumption (A18) leads to a contradiction. Consequently, $\varphi(x) > \varphi_0$ and $v_k/m > v_{k0}/m_0$, or

$$m < \frac{m_0}{v_{k0}} v_k. \tag{A19}$$

However, as follows from (A9), at $\alpha > 1$

$$\lim_{m \to +0} v_k = 0. \tag{A20}$$

It follows from (A19) and (A20) that $\lim\limits_{v_k \to +0} m = 0$, i.e., that, given conditions (A11), the mass of the droplet and its velocity vanish simultaneously.

APPENDIX B. DERIVATION OF EQ. (16)

At $h \to 0$, from (2)-(9) we obtain

$$\frac{dm}{dx} = -4\pi \frac{\lambda}{\varkappa} \frac{r}{v_k} \Delta T, \quad \frac{dv_k}{dx} = -6\pi\eta \frac{r}{m}, \tag{B1}$$

where $\Delta T = T - T_s \approx (\Delta T_L/L)x$.

It follows from (B1) that

$$\frac{m_0}{r_0^3} \frac{d(r^2 v_k)}{dx} = -\left(\frac{8\pi\lambda}{3\varkappa} \Delta T + 6\pi\eta\right). \tag{B2}$$

If v_k vanishes at the point $x = L$, then integration of (B2) within the interval from $x = 0$ to $X = L$ yields

$$\frac{m_0 v_{k0}}{r_0^2} = \frac{8\pi\lambda}{3\varkappa} \frac{\Delta T_L}{2} L + 6\pi\eta L,$$

from which Eq. (16) directly follows.

NOTATION

R, universal gas constant; c_V, specific heat of gas at constant volume; $c_p = c_V + R$; λ, thermal conductivity of gas; $\varkappa$, heat of vaporization of liquid; T_s, saturation temperature; η, viscosity coefficient of gas.

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FLOW BENEATH AN INCLINED PLATE, ESTABLISHED IN AN AGGREGATIVELY UNSTABLE SYSTEM

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The flow of a film under a plate is investigated. An expression is obtained for determining the angle of inclination of the plate, for large values of which the flow of the film transforms into wave motion.

Gravitational separation of aggregatively unstable systems, represented by highly concentrated finely dispersed particles, in the space between the inclined elements (plates, pipes) proceeds more vigorously than settling in an infinite space. Many workers have shown that under static and dynamic conditions the maximum rate of the sedimentation process for suspensions with concentrations 10-50 kg/m^3 is observed for plates positioned at an angle of 30-40° to the horizontal [1, 2]. It was assumed that the basic factor that influences the increase in the sedimentation rate of suspensions is the smallest height of the sediment determined along the vertical from the upper plate to the lower plate. However, as experience has shown, separation of an aggregatively unstable system in the space between inclined plates is accompanied by the formation of a film flow of the clarified liquid under the upper plate, which must have an effect on the sedimentation rate. We studied the flow that forms on an experimental setup which consisted of a collection of flat glass tubes closed at one end ($l = 0.5$ and 1.0 m; $b = 0.15$ m; $h = 0.023$, 0.033, and 0.053 m), a device for recording the changes in the velocity of motion of the film, including a MT-54M microthermistor sensor and a plotting device, and a system for photographing different aspects of the flow. The modeling medium was chosen as an unflocculated suspension of kaolin with a density of $\rho_s = 1010\text{-}1015$ kg/m^3. The investigations were carried out under static conditions using the following technique. The flat glass tubes were filled with carefully mixed modeling medium and they were placed successively at different angles to the horizontal. The sedimentation process in the inclined tube was accompanied by the formation of a film flow of the clear liquid beneath the upper plate. A coloring liquid was introduced into the effective cross section of the moving film in order to make visual observations and a microthermistor sensor was also inserted in order to record changes in the velocity of motion of the film.

When the angle of inclination of the tube was varied from 20 to 80°, visual analysis of the motion of the colored liquid revealed the presence of two regimes in the flow of the film: laminar and turbulent; analysis of the trace on the plotter in juxtaposition to the motion of the colored stream indicated the presence of three regimes: stable laminar, laminar with the formation of waves, and turbulent. As an example, Fig. 1 shows the stable laminar regime (a) and the regime with clearly manifested turbulence (b). Figure 2 shows typical chromograms of the three flow regimes of the thin clarified liquid film. When the tubes are placed at an angle to the horizontal less than 30°, the colored stream and the trace on the plotter have an identical character: a smooth line without perturbations. At an angle of inclination approximately equal to 40°, the flow of the colored stream shows the continued presence of laminar film motion, while simultaneously the chromogram reveals pulsations in the velocity and waves are visually observed, i.e. the motion of the film, while remaining laminar, acquires a wave character. When the tubes are placed at an angle exceeding 50° to the horizontal, the flow of the film is clearly turbulent, the streamlike nature of the colored liquid is destroyed, and eddies and large-scale vortex zones appear (Fig. 1b). On the chromogram in Fig. 2c, velocities of motion of the film differing by factors of 2-3 and more are distinguished. The film thickness was determined by measuring

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